

- (1) Work the subtraction problem  $82 - 47$  in the three ways discussed in class. In each case, explain carefully what you are doing and why it will give you the correct answer.
- (2) Work the subtraction problem  $\star\heartsuit - \Delta\square$  in the three ways discussed in class. In each case, explain carefully what you are doing and why it will give you the correct answer.
- (3) Write a method for someone unfamiliar with happy numbers on how to convert every possible happy number to an Arabic number. Show how your method works by converting  $\heartsuit\Delta\heartsuit\text{D}$  to Arabic. Be sure to show where each number comes from in your method.
- (4) People have devised many kinds of secret codes to have private communications with each other, so that no one else can understand their messages. We will consider a letter-number code, where arithmetic problems are disguised by using letters. Let's look at an example:

$$\begin{array}{r} 35 \\ + 35 \\ \hline 70 \end{array} \qquad \begin{array}{r} AD \\ + AD \\ \hline NH \end{array}$$

To create a coded version of the problem, you replace each digit with a letter, always using the same letter for that particular digit. For example, we replaced 3 with A, 5 with D, 7 with N, and 0 with H in the above example.

Here are the rules for creating a code:

- (a) In the same problem, a letter always stands for the same number each time it is used.
- (b) Different letters in the same problem always stand for different single digit numbers.
- (c) A letter standing for 0 never starts a number with more than one digit. For example, the problem can't have a number like "05" (but it can use "507" or "80" or simply "0".)

See whether you can crack the codes for the following problems. If you think there is only one right answer, prove it—that is, give a good explanation why you think so. If you think there are several possibilities, give them all and prove that there are no others. If you think there is no possible answer, prove that.

$$\begin{array}{r} ABB \\ - \quad A \\ \hline DD \end{array} \qquad \begin{array}{r} EE \\ + \quad FF \\ \hline EEK \end{array} \qquad \begin{array}{r} PA \\ + \quad AP \\ \hline PUG \end{array}$$

- (5) Write an essay about your experience with the three subtraction schemes we discussed this week. What are the advantages and disadvantages of each method? Which one is your favorite? Which one(s) would you use in your own classroom?