MATH 310 Homework Quiz 1 (4 September 2024)

(a) Let a and b be integers. Define $a|b$.

(b) Write the numbers twenty-five, thirty-two, and fifty-six to the base five.

MATH 310 Homework Quiz 2 (11 September 2024)

(a) Let a and b be integers. Define $\gcd(a, b)$.

(b) Find the greatest common divisor of 108 and 243.

MATH 310 Homework Quiz 3 (18 September 2024)

(a) Define a prime number.

(b) Find the following least common multiples:

(i) $\text{lcm}(125, 150)$

(ii) $\text{lcm}(p^2q, pqr)$ where p, q, and r are primes.

MATH 310 Homework Quiz 4 (25 September 2024)

(a) Define $a \equiv b \bmod m$.

(b) Find integers x such that $7x \equiv 6 \bmod 5$.

MATH 310 Homework Quiz 5 (2 October 2024)

(a) Define $\phi(m)$.

(b) What is the remainder when 41^{75} is divided by 3?

MATH 310 Homework Quiz 6 (9 October 2024)

(a) Define what it means for a function f to be multiplicative.

(b) Prove that $\phi(m)$ is even if $m > 2$.

MATH 310 Homework Quiz 7 (23 October 2024)

(a) Define a primitive root mod n.

(b) Prove that $\displaystyle\sum_{d|n} \mu(d)\,\phi(d) = \prod_{p|n}(2-p)\,.$

MATH 310 Homework Quiz 8 (30 October 2024)

(a) Are there infinitely primes of the form $4k+3$? (No explanation necessary.)

(b) Prove that each odd primitive root modulo p^m (p and odd prime) is a primitive root modulo $2p^m$.

MATH 310 Homework Quiz 9 (6 November 2024)

(a) Define the Legendre symbol $(\frac{a}{p})$.

(b) What is the smallest positive integer x for which $x^2 - x + 41$ is not a prime? (No explanation necessary.)

MATH 310 Homework Quiz 10 (13 November 2024)

(a) Start the continued fraction expansion of $\frac{100}{37}$ (the first two terms suffice).

(b) Does $3x^2 \equiv 12 \bmod 23$ have a solution?

MATH 310 Homework Quiz 11 (20 November 2024)

(a) Which number is represented by the continued fraction $[1; 1, 1, 1, \dots]$? (No explanation necessary).

(b) Determine the infinite continued fraction of $\frac{1+\sqrt{13}}{2}$.

MATH 310 Homework Quiz 12 (4 December 2024)

(a) What is the generating function for the number of partitions with parts 2 and 5? (No explanation necessary).

(b) Form the graphical representation of the partition $8+6+2+2+1$ and find the conjugate partition.

MATH 310 Homework Quiz 13 (11 December 2024)

(a) What is the generating function for the number of partitions with distinct parts? (No explanation necessary).

(b) Let $d_m(n)$ denote the number of partitions of n into distinct parts $\leq m$, with generating function

$$L_m(q) = \sum_{n=0}^{\infty} d_m(n)\, q^n.$$

Prove that $L_m(q) = L_{m-1}(q) + q^m\, L_{m-1}(q)\,.$