## San Francisco Math Circles - Sudoku

September/October 2006

|  |  |  | 6 |  |  |  |  | 5 |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 5 |  | 6 |  |  |
|  | 5 |  |  | 8 |  |  |  | 9 |
|  | 2 |  |  | 1 | 6 | 7 |  |  |
|  |  | 1 | 7 |  |  | 9 | 6 |  |
| 7 | 6 |  |  | 2 |  |  |  | 3 |
|  | 9 |  | 3 |  |  |  | 2 | 7 |
|  | 3 | 7 |  | 4 |  |  |  | 6 |
| 6 |  |  | 9 |  | 1 | 3 |  |  |


|  | 1 |  |  |  |  |  |  | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 5 |  |  |  |  |  | 7 |  |
|  |  | 7 |  | 3 | 8 |  |  | 1 |
|  |  |  |  | 5 |  | 4 |  |  |
| 5 | 4 |  | 1 |  |  | 3 |  |  |
|  |  |  |  | 4 |  |  | 1 |  |
| 3 | 7 |  |  | 9 | 2 |  | 8 | 6 |
|  |  |  |  |  |  |  |  |  |
| 8 | 6 |  |  | 1 | 4 | 5 | 3 |  |

A Sudoku square is a $9 \times 9$ grid filled with nine symbols (such as the numbers from 1 to 9 ) in such a way that each row, column, and the nine $3 \times 3$ subsquares (shown above) contain each symbol exactly once. The two squares above contain some entries (called clues), and your task is to complete the Sudoku squares. It turns out that there is only one way to complete each square. (Caveat: If you've never played a Sudoku puzzle, watch out-these squares are addictive.) We will work on two problems regarding Sudoku squares:
(1) How many Sudoku squares are there?
(2) What is the minimum number of clues that yield a unique solution to a Sudoku puzzle?

These are hard questions. In fact, (1) was answered only last year, and (2) remains open. So we will simplify the problems and work with $4 \times 4$ Sudoku squares.

