

Name: \_\_\_\_\_

Show complete work—that is, all the steps needed to completely justify your answer. Simplify your answers as much as possible. You may refer to theorems that we proved in class.

- (1) (a) Define the conjugate  $\bar{z}$  of the complex number  $z = x + iy$ .
- (b) Show that  $z$  is purely imaginary (i.e, has real part 0) if and only if  $z = -\bar{z}$ .

- (2) (a) Define the polar coordinates of the complex number  $z = x + iy$ .
- (b) Find all solutions to  $z^4 = -16$ .

(3) (a) Define  $\lim_{z \rightarrow z_0} f(z) = w_0$ .

(b) Prove that  $\lim_{z \rightarrow z_0} f(z) = 0$  if and only if  $\lim_{z \rightarrow z_0} |f(z)| = 0$ .

You will be allowed to (once) revise and resubmit Problems 2(b) and 3(b) by the beginning of class on 2/26/25. For the revision, you are not allowed to communicate with your classmates, and you may use neither internet nor AI sources. I will reserve the right to ask you about your work if I suspect that you violated these rules.