MATH 380 Exam 2 3/10/25 10:00–10:50 a.m.

Name: ________________________________

Show complete work—that is, all the steps needed to completely justify your answer. Simplify your answers as much as possible. You may refer to theorems that we proved in class.

(1) (a) Define what it means for a function $f(z)$ to be holomorphic at z_0.

(b) Prove: If $f(z)$ is entire and real valued (that is, $\text{Im}(f(z)) = 0$ for all $z \in \mathbb{C}$) then $f(z)$ is constant.

(2) (a) Define the complex exponential function.

(b) For each of the following functions, give the (maximal) subset of $\mathbb{C}$ where it is differentiable. Choose one of the functions and compute its derivative.

(i) $f(z) = z^{3i}$

(ii) $f(z) = (3i)^z$

(3) (a) Define $\int_\gamma f(z)\,dz$ for a curve $\gamma : [a,b] \to \mathbb{C}$.

(b) Compute $\int_\gamma \frac{1}{z^2}\,dz$ where γ is the unit circle, oriented counterclockwise.

You will be allowed to (once) revise and resubmit Problem 2(b) by the beginning of class on 3/17/25. For the revision, you are not allowed to communicate with your class mates, and you may use neither internet nor AI sources. I will reserve the right to ask you about your work if I suspect that you violated these rules.