MATH 380 Exam 3 4/7/25 10:00–10:50 a.m.

Name: _____

Show complete work—that is, all the steps needed to completely justify your answer. Simplify your answers as much as possible. You may refer to theorems that we proved in class.

(1) (a) Define a homotopy and what it means that γ_0 is G-homotopic to γ_1.

(b) Compute $\int_\gamma \frac{\sin z}{z-1}\,dz$, where γ is the counterclockwise circle centered at the origin with radius 2.

(2) (a) Define what it means for a closed curve γ to be G-contractible.

(b) Compute $\int_\gamma \frac{\sin z}{z-3}\,dz$, where γ is the counterclockwise circle centered at the origin with radius 2.

(3) (a) Let γ be a smooth curve. Define its length.

(b) Suppose f is entire and there exists $M > 0$ such that $|f(z)| \geq M$ for all $z \in \mathbb{C}$. Prove that f is constant.

You will be allowed to (once) revise and resubmit Problems 2(b) and 3(b) by the beginning of class on 4/16/25. For the revision, you are not allowed to communicate with your class mates, and you may use neither internet nor AI sources. I will reserve the right to ask you about your work if I suspect that you violated these rules.