

Name: \_\_\_\_\_

Show complete work—that is, all the steps needed to completely justify your answer. Simplify your answers as much as possible. You may refer to theorems that we proved in class.

(1) (a) Define  $\lim_{n \rightarrow \infty} a_n = L$ .

(b) Prove convergence/divergence of the sequence  $a_n = \frac{(-1)^n}{n}$ .

- (2) (a) Define what it means for a series to converge absolutely.
- (b) Find the power series about the origin for the function  $f(z) = \cos(z^2)$  and determine its radius of convergence.

- (3) (a) Define (1) pointwise and (2) uniform convergence of the function sequence  $f_n(z)$  converging to  $f(z)$ .
- (b) Compute the limit function of the power series  $\sum_{k \geq 0} 2^k (z-2)^k$  and determine where it converges absolutely and where it converges uniformly.

You will be allowed to (once) revise and resubmit Problems 2(b) and 3(b) by the beginning of class on 5/7/25. For the revision, you are not allowed to communicate with your classmates, and you may use neither internet nor AI sources. I will reserve the right to ask you about your work if I suspect that you violated these rules.