

**MATH 420/720      Poset Exercises**

- (1) Given a poset with  $mn + 1$  elements, show that there is a chain of size  $m + 1$  or an antichain of size  $n + 1$ .
- (2) Given a finite poset  $(P, \preceq)$ , let

$$\eta(x, y) := \begin{cases} 1 & \text{if } x \prec y, \\ 0 & \text{otherwise.} \end{cases}$$

Prove that  $\eta^k(x, y)$  equals the number of chains of length  $k$  starting in  $x$  and ending in  $y$ .

- (3) Let  $P = [12]$  under the divisibility relation. Compute  $\zeta$  and  $\mu$  for this poset.
- (4) (a) Let  $P = [d]$  with the usual order of integers. Show that for  $1 \leq i < j \leq d$

$$\mu(i, j) = \begin{cases} 1 & \text{if } i = j, \\ -1 & \text{if } i + 1 = j, \\ 0 & \text{otherwise.} \end{cases}$$

- (b) Write out the statement that Möbius inversion gives in this explicit case and interpret it along the lines of the Fundamental Theorem of Calculus.