MATH 420/720 Ramsey Exercises

(1) Prove that $R(m,n) \leq \binom{m+n-2}{m-1}$.

(2) Show that $R(3,5) = 14$.

(3) Show that $R(4,4) = 18$.

(4) Suppose $a := R(m-1,n)$ and $b := R(m,n-1)$ are both even, and let $G = (V,E)$ be a simple graph with $a+b-1$ vertices that has neither an m-clique nor an independent n-set.

(a) Show that the degree of any vertex is less than a. (One way to prove this is to assume, by contradiction, that $v \in V$ has degree $\geq a$, and consider the subgraph induces by all neighbors of v.)

(b) We recall that the *complement* of G has the same vertex set and the complementary edge set. Show that the degree of any vertex in the complement of G is less than b.

(c) Deduce that every vertex of G has degree $a-1$ and show (e.g., by considering the number of edges in G) that this leads to a contradiction.

Note that this proves that $R(m,n) < R(m-1,n) + R(m,n-1)$ if both terms on the right are even.

(5) Prove that $R(3,n) \leq \frac{n^2+3}{2}$. (*Hint:* consider the parity of n as separate cases.)