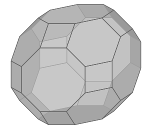
MATH 883: POLYTOPES & VARIETIES
Spring 2022



Homework V
due 4 March

(1) A finite poset $(\Phi, \preceq)$ in which the meet of any two elements exists is called a *meet semilattice*. Show that if a meet semilattice $(\Phi, \preceq)$ has a maximum element, then it is a lattice.

(2) Give a complete description of the faces of $[-1,1]^d$ and their face lattice.

(3) A polytope P is *simplicial* if every proper face of P is a simplex. Show that a cross polytope is simplicial.

(4) (**Collaborative**) Let $P \subset \mathbf{R}^d$ be a polytope of dimension $< d$. For a point $\mathbf{v} \in \mathbf{R}^d \setminus \operatorname{aff}(P)$, we define the pyramid[1]

$$\mathbf{v} * P \ := \ \operatorname{conv}(P \cup \{\mathbf{v}\}) .$$

Prove that for each face $F \preceq P$, the polytope $\mathbf{v} * F$ is a face of $\mathbf{v} * P$. Conversely, show that every face of $\mathbf{v} * P$ is either a face of P or of the form $\mathbf{v} * F$ for some face $F \preceq P$.

[1] One can show that $\mathbf{v} * P$ is linearly isomorphic to $\mathbf{w} * P$ for any other $\mathbf{w} \in \mathbf{R}^d \setminus \operatorname{aff}(P)$, which justifies calling this *the* pyramid over P.