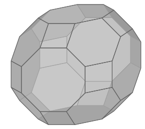
MATH 883: POLYTOPES & VARIETIES
Spring 2022



Homework X
due 15 April

(1) Compute the integer-point transform $\sigma_C(\mathbf{z})$ for the following (affine) cones:
  (a) $C = \{\lambda_1(0,1) + \lambda_2(1,0) : \lambda_1, \lambda_2 \geq 0\}$;
  (b) $C = \{\lambda_1(0,1) + \lambda_2(1,1) : \lambda_1, \lambda_2 \geq 0\}$;
  (c) $C = \{(3,4) + \lambda_1(0,1) + \lambda_2(2,1) : \lambda_1, \lambda_2 \geq 0\}$.

(2) Compute the Ehrhart polynomials and the Ehrhart series of the simplices with the following vertices:
  (a) $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$;
  (b) $(0,0,0,0)$, $(1,0,0,0)$, $(0,2,0,0)$, $(0,0,3,0)$, and $(0,0,0,4)$.

(3) A lattice simplex $\Delta = \operatorname{conv}\{\mathbf{v}_0, \mathbf{v}_1, \ldots, \mathbf{v}_d\} \subset \mathbf{R}^d$ is *unimodular* if $\mathbf{v}_1 - \mathbf{v}_0, \mathbf{v}_2 - \mathbf{v}_0, \ldots, \mathbf{v}_d - \mathbf{v}_0$ is a lattice basis of $\mathbf{Z}^d$, i.e., every vector in $\mathbf{Z}^d$ can be uniquely written as an integer combination of $\mathbf{v}_1 - \mathbf{v}_0, \mathbf{v}_2 - \mathbf{v}_0, \ldots, \mathbf{v}_d - \mathbf{v}_0$. Compute the Ehrhart polynomial of Δ and conclude that $\operatorname{vol}\Delta = \frac{1}{d!}$.

(4) Let $\Delta \subset \mathbf{R}^d$ be a unimodular d-simplex and $\mathbf{q}$ a point generic relative to Δ. Prove that

$$h^*_{\mathbb{H}_{\mathbf{q}}\Delta}(z) \;=\; z^r$$

where r is the number of visible facets in $\mathbb{H}_{\mathbf{q}}\Delta$.