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AG Diskrete Geometrie
Arnimallee 2

Discrete Geometry I
Wintersemester 2020/21

Take Home Final Exam II 6–8 April 2021

Show complete work—that is, all the steps needed to completely justify your answer. Simplify your answers as much as possible. You may refer to theorems in the text books. Each of the problems will be weighted equally.

You are welcome to use books and internet sources, but you are not allowed to discuss this exam with anyone (this includes live discussions, calls, chats, etc.). I reserve the right for an oral follow-up meeting if I suspect that you did not follow these rules.

The exam is due on at 12 noon on 8 April 2021 (FU Whiteboard or email), and your submission should be a pdf file (typed or carefully scanned; if you do not have easy access to a scanner, send me a photo by the deadline and a detailed scan by 6pm on 11 March 2021). Please include this page with your name and Matrikelnummer added below, as an indication that you understand and agree with the rules for this exam.

____________________ Name

____________________ Matrikelnummer

Show complete work—that is, all the steps needed to completely justify your answer. Simplify your answers as much as possible. You may refer to theorems in the text books. You may use software for computations; in this case, tell me what are you computing and underline mathematically that it gives you what you're trying to prove. Each of the problems will be weighted equally.

1. Let $P := \left\{\mathbf{x} \in \mathbb{R}^d : x_1 + x_2 + \cdots + x_d = 2,\ 0 \le x_j \le 1 \text{ for all } 1 \le j \le d\right\}$.
   (a) Compute the vertices of P.
   (b) Compute the signed cocircuits of the oriented matroid defined by P and deduce its facets.
2. Let $P = [0,1]^3$, the 3-dimensional unit cube. Compute the regular triangulation of P induced by the height function $w(\mathbf{v}) := (v_1 + v_2 + v_3)(3 - v_1 - v_2 - v_3)$.
3. Let $P \subset \mathbb{R}^d$, $Q \subset \mathbb{R}^e$ be two polyhedra.
   (a) Show that $P \times Q$ is a polyhedron.
   (b) Show that $\Phi(P \times Q) \setminus \{\varnothing\}$ and $(\Phi(P) \setminus \{\varnothing\}) \times (\Phi(Q) \setminus \{\varnothing\})$ are isomorphic as posets.
   (c) Describe the face structure of a prism over a given polyhedron P.
4. Consider a d-dimensional arrangement consisting of the hyperplanes $H_0, H_1, \ldots, H_k$ with the property that the intersection of $j \le d$ hyperplanes has dimension $d - j$, and the intersection of $j > d$ hyperplanes is empty if H_0 is one of the hyperplanes, and of dimension 0 otherwise. Compute the characteristic polynomial of this arrangement and deduce its number of regions.