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AG Diskrete Geometrie
Arnimallee 2

Discrete Geometry III
Wintersemester 2020/21

Take Home Final Exam II 6–8 April 2021

Show complete work—that is, all the steps needed to completely justify your answer. Simplify your answers as much as possible. You may refer to theorems in the text book. Each of the problems will be weighted equally.

You are welcome to use books and internet sources, but you are not allowed to discuss this exam with anyone (this includes live discussions, calls, chats, etc.). I reserve the right for an oral follow-up exam if I suspect that you did not follow these rules.

The exam is due on at 12 noon on 8 April 2021 (FU Whiteboard or email), and your submission should be a pdf file (typed or carefully scanned). Please include this page with your name and Matrikelnummer added below, as an indication that you understand and agree with the rules for this exam.

Name ______________________ Matrikelnummer ______________________

Show complete work—that is, prove all steps needed to completely justify your answer.

1. Let H be a d-dimensional hyperplane arrangement consisting of k hyperplanes with the property that the intersection of any j of the hyperplanes has dimension $d - j$, and vice versa, every $(d-j)$-dimensional flat of H is the intersection of j hyperplanes, for $2 \le j \le d$. Compute the characteristic polynomial of H and deduce its number of regions.

2. Let Π be the Boolean lattice on d elements. Compute its zeta polynomial.

3. Let P be the d-dimensional cross polytope, i.e., the convex hull of the standard unit vectors and their negatives. For each facet F of P, let $S(F)$ be the convex hull of F and the origin. Prove that the set of all such $S(F)$ and their faces form a regular triangulation of P.

4. Given a unimodular d-simplex Δ and a point $\mathbf{q}$ generic relative to Δ, compute $h^*_{\mathbb{H}^{\mathbf{q}}\Delta}(z)$.