

[Problem#] Proposed by Matthias Beck, Jesus DeLoera, Mike Develin, and Julian Pfeifle, MSRI, Berkeley, CA. Given $t_1, t_2, \dots, t_d \in \mathbb{Z}$ and $\lambda_1, \lambda_2, \dots, \lambda_d \in \mathbb{R}$, prove that

$$\forall 1 \leq j < d : \sum_{k=1}^d \lambda_k t_k^j \in \mathbb{Z} \quad \implies \quad \sum_{k=1}^d \lambda_k t_k^d \in \mathbb{Z}.$$