Homework Problems MATH 227 (Calculus II) Spring 2005

Evaluate the integral.

1. $\int (2-x)^6 dx$ 2. $\int \frac{1+4x}{\sqrt{1+x+2x^2}}\, dx$ 3. $\int \frac{1}{(5t+4)^{2.7}}\, dx$
4. $\int \sqrt{4-t}\, dt$ 5. $\int \cot x\, dx$ 6. $\int \sin^3 x\, \cos x\, dx$
7. $\int \tan^2\theta\, \sec^2\theta\, d\theta$ 8. $\int_0^{\pi/3} \frac{\sin\theta}{\cos^2\theta}\, d\theta$ 9. $\int_0^4 \frac{dx}{(x-2)^3}$

Find the area enclosed by the given curves.

10. $y = 12 - x^2$, $y = x^2 - 6$
11. $y = x^3 - x$, $y = 3x$
12. $y = \sqrt{x}$, $y = x/2$,$x = 9$
13. $y = \sin(\pi x/2)$, $y = x$
14. $y = \cos x$, $y = \sin 2x$, $x = 0$, $x = \pi/2$
15. $y = \sin x$, $y = \sin 2x$, $x = 0$, $x = \pi/2$
16. $y = \cos x$, $y = 1 - 2x/\pi$
17. $y = |x|$, $y = x^2 - 2$
18. $y = \sin \pi x$, $y = x^2 - x$, $x = 2$

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specific line. Sketch the region and the solid.

19. $y = x^2, y^2 = x$; about the x-axis
20. $y = \sec x, y = 1, x = -1, x = 1$; about the x-axis
21. $y^2 = x, x = 2y$; about the y-axis
22. $y = x^{2/3}, x = 1, y = 0$; about the y-axis
23. $y = x, y = \sqrt{x}$; about $y = 1$
24. $y = x^2, y = 4$; about $y = 4$
25. $y = x^4, y = 1$; about $y = 2$

Each integral represents the volume of a solid. Describe the solid.

26. $\pi \int_0^{\pi/2} \cos^2 x\, dx$ 27. $\pi \int_2^5 y\, dy$

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specific line. Sketch the region and the solid.

28. $y = x^3, y = 8, x = 0$; about the x-axis
29. $x = 4y^2 - y^3, x = 0$; about the x-axis
30. $y = 4x^2, 2x + y = 6$; about the x-axis
31. $x + y = 3, x = 4 - (y-1)^2$; about the x-axis
32. $y = x^2, y = 0, x = 1, x = 2$; about $x = 1$
33. $y = x^2, y = 0, x = -2, x = -1$; about the y-axis
34. $y = x^2, y = 0, x = 1, x = 2$; about $x = 4$

Each integral represents the volume of a solid. Describe the solid.

35. $\int_0^3 2\pi x^5\, dx$ 36. $2\pi \int_0^2 \frac{y}{1+y^2}\, dy$

Compute the integral.

37. $\int x \cos 5x\, dx$ 38. $\int x e^{-x} dx$ 39. $\int r e^{r/2} dr$ 40. $\int t \sin 2t\, dt$
41. $\int \ln(2x+1)\, dx$ 42. $\int \sin^{-1} x\, dx$ 43. $\int \arctan 4t\, dt$

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specific line. Sketch the region and the solid.

44. $y = e^x, y = e^{-x}, x = 1$; about the y-axis

45. $y = e^{-x}, y = 0, x = -1, x = 0$; about $x = 1$

Compute the integral.

46. $\displaystyle\int \frac{x-9}{(x+5)(x-2)}\,dx$ 47. $\displaystyle\int \frac{1}{(t+4)(t-1)}\,dt$ 48. $\displaystyle\int_2^3 \frac{1}{x^2-1}\,dx$

49. $\displaystyle\int_0^1 \frac{x-1}{x^2+3x+2}\,dx$ 50. $\displaystyle\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy$ 51. $\displaystyle\int \frac{x^2+2x-1}{x^3-x}\,dx$

52. $\displaystyle\int_0^1 \frac{2x+3}{(x+1)^2}\,dx$ 53. $\displaystyle\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\,dx$ 54. $\displaystyle\int \frac{dx}{(x+5)^2(x-1)}$

55. $\displaystyle\int \frac{x^2}{(x-3)(x+2)^2}\,dx$ 56. $\displaystyle\int \frac{dx}{x-\sqrt{x+2}}$ 57. $\displaystyle\int \frac{\sqrt{x}}{x-4}\,dx$

Hint: In the last two problems, make a substitution to express the integral as a rational function.

Approximate the following integrals by Taylor sums using left-, right-endpoints, and midpoints, using 20 subintervals.

58. $\displaystyle\int_0^1 e^{x^2}\,dx$ 59. $\displaystyle\int_1^2 \frac{\cos x}{x}\,dx$

Compute the integral.

60. $\displaystyle\int_4^\infty e^{-y/2}\,dy$ 61. $\displaystyle\int_{-\infty}^{-1} e^{-2t}\,dt$ 62. $\displaystyle\int_{-1}^0 \frac{dx}{x^2}$ 63. $\displaystyle\int_9^{34} \frac{dx}{\sqrt[3]{x-9}}$

Find the length of the curve.

64. $y = \dfrac{x^5}{6} + \dfrac{1}{10x^3},\ 1 \le x \le 2$ 65. $y = \dfrac{x^2}{2} - \dfrac{\ln x}{4},\ 2 \le x \le 4$

66. $y = \ln(\cos x),\ 0 \le x \le \pi/3$ 67. $y = \ln x,\ 1 \le x \le \sqrt{3}$

Find the area of the surface obtained by rotating the curve about the x-axis.

68. $y = \sqrt{x},\ 4 \le x \le 9$ 69. $y = \cosh x,\ 0 \le x \le 1$ 70. $y = \dfrac{x^3}{6} + \dfrac{1}{2x},\ \dfrac{1}{2} \le x \le 1$

Find the area of the surface obtained by rotating the curve about the y-axis.

71. $y = 1 - x^2,\ 0 \le x \le 1$

Sketch the curves given by the following parametric equations. Eliminate the parameter to find a Cartesian equation for each curve.

72. $x = t^2 - 2,\ y = 5 - 2t,\ -3 \le t \le 4$ 73. $x = 1 + 3t,\ y = 2 - t^2$

74. $x = 4\cos\theta,\ y = 5\sin\theta,\ -\pi/2 \le \theta \le \pi/2$ 75. $x = \ln t,\ y = \sqrt{t},\ t \ge 1$

Find an equation of the tangent(s) to the curve at the given point. Then graph the curve and the tangent(s).

76. $x = 2\sin 2t,\ y = 2\sin t;\ \left(\sqrt{3}, 1\right)$ 77. $x = \sin t,\ y = \sin(t + \sin t);\ (0,0)$

Find the the points on the curve where the tangent is horizontal or vertical.

78. $x = 10 - t^2,\ y = t^3 - 12t$ 79. $x = 2t^3 + 3t^2 - 12,\ y = 2t^3 + 3t^2 + 1$

Find the area bounded by the curve.

80. $x = t - 1/t,\ y = t + 1/t$ and the line $y = 2.5$.

81. $x = \cos t,\ y = e^t,\ 0 \le t \le \pi/2$ and the lines $y = 1$ and $x = 0$.

Find the length of the curve.

82. $x = e^t + e^{-t},\ y = 5 - 2t,\ 0 \le t \le 3$

83. $x = e^t - t,\ y = 4\,e^{t/2},\ -8 \le t \le 3$

Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions.

84. $0 \le r < 4,\ -\pi/2 \le \theta < \pi/6$ 85. $2 < r \le 5,\ 3\pi/4 < \theta < 5\pi/4$

Identify the curve by finding a Cartesian equation for the curve.

86. $r\cos\theta = 1$ 87. $r = 3\sin\theta$

Find a polar equation for the curve represented by the given Cartesian equation.

88. $x = -y^2$ 89. $x + y = 9$

Sketch the curve with the given polar equation.

90. $r = \sin 2\theta$ 91. $r = 2\cos 3\theta$

Determine whether the sequence converges or diverges. If it converges, find the limit.

92. $a_n = n(n-1)$ 93. $a_n = \dfrac{n+1}{3n-1}$ 94. $a_n = \dfrac{2^n}{3^{n+1}}$

95. $a_n = \dfrac{n}{1+\sqrt{n}}$ 96. $a_n = \dfrac{\cos^2 n}{2^n}$ 97. $a_n = \dfrac{(-3)^n}{n!}$

Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

98. $a_n = \dfrac{2n-3}{3n+4}$ 99. $a_n = n + \dfrac{1}{n}$

Determine whether the series converges or diverges. If it converges, find its sum.

100. $\dfrac{1}{8} - \dfrac{1}{4} + \dfrac{1}{2} - 1 + \cdots$ 101. $-2 + \dfrac{5}{2} - \dfrac{25}{8} + \dfrac{125}{32} + \cdots$ 102. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

103. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$ 104. $\displaystyle\sum_{k=1}^{\infty} (\cos 1)^k$ 105. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n}\right)$

Find the value of x for which the series converges. Find the sum of the series for those values of x.

106. $\displaystyle\sum_{n=0}^{\infty} 4^n x^n$ 107. $\displaystyle\sum_{n=0}^{\infty} \frac{(x+3)^n}{2^n}$

Determine whether the series converges or diverges.

108. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}$ 109. $\displaystyle\sum_{n=1}^{\infty} e^{-n}$ 110. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ 111. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2-4n+5}$

112. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ 113. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$ 114. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-5n}{n^3+n+1}$ 115. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+7n}{3^n\,(n^2+5n-1)}$

116. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2n}{4n^2+1}$ 117. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+2\sqrt{n}}$ 118. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n!}$ 119. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

120. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$ 121. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$ 122. $\displaystyle\sum_{n=1}^{\infty} \frac{n(-3)^n}{4^{n-1}}$ 123. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2\,2^n}{n!}$

Find the radius of convergence and interval of convergence of the series.

124. $\sum_{n=0}^{\infty} n^3(x-5)^n$ 125. $\sum_{n=0}^{\infty} \frac{(3x-2)^n}{n\,3^n}$

Find a power series representation for the function and determine the interval of convergence.

126. $f(x) = \frac{1}{1+9x^2}$ 127. $f(x) = \frac{x}{4x+1}$ 128. $f(x) = \ln(5-x)$ 129. $f(x) = \frac{x^3}{(x-2)^2}$

Find the Taylor series for f centered at a.

130. $f(x) = \cos x\,,\ a = \pi$ 131. $f(x) = x^{-2}\,,\ a = 1$