Proximity Graphs

- A proximity graph is a simply a graph in which two vertices are connected by an edge if and only if the vertices satisfy particular geometric requirements.

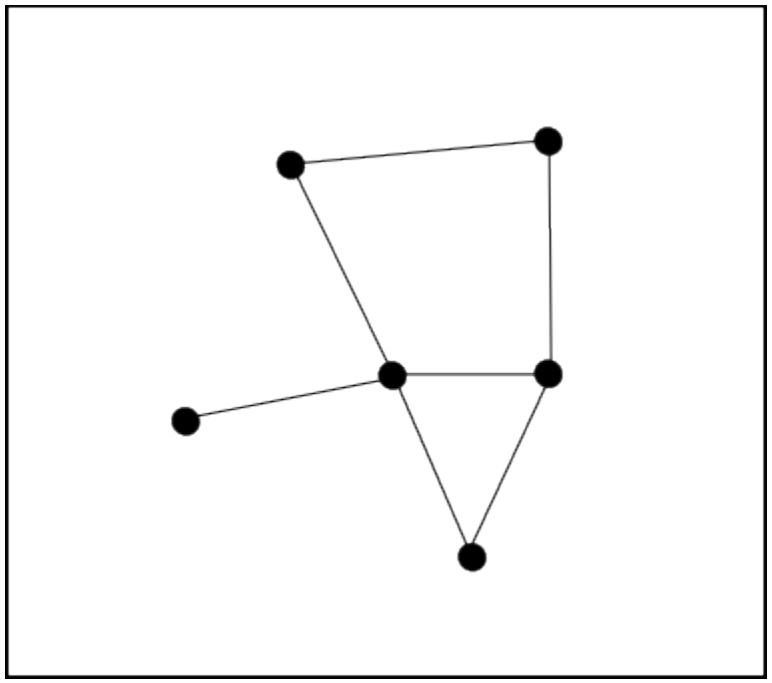
- “Proximity” here means spatial distance.

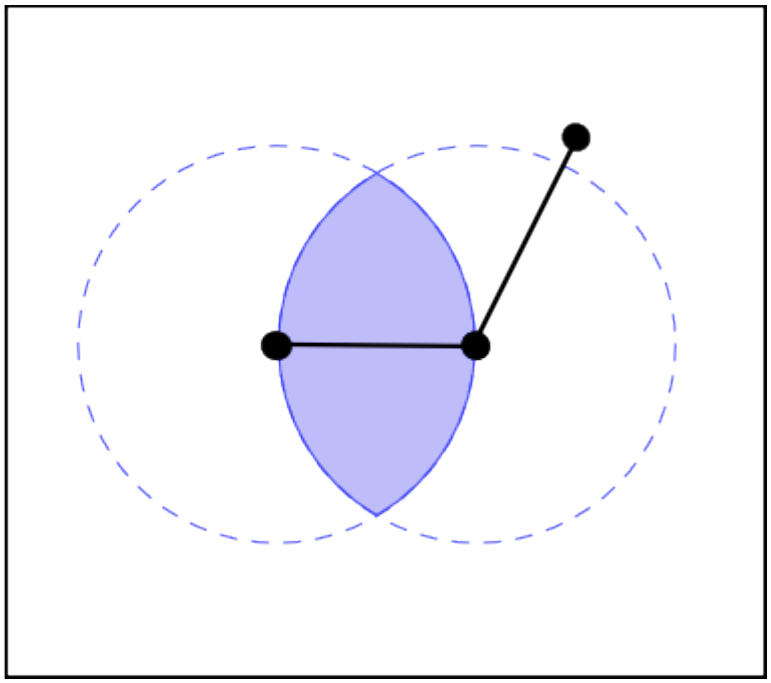
- Many of these graphs can be formulated with respect to many metrics, but the Euclidean metric is used most frequently.

Relative Neighborhood Graphs

- Let $\Lambda(p,q)$ be the intersection of the circle about p with a radius of dist(p,q) and the circle about q with a radius of dist(q,p). This is called a *lune*.

- The *relative neighborhood graph RNG(V)* of a set of points V, is the graph that has an edge (p,q) if and only if the intersection of $\Lambda(p,q)$ and V is empty.

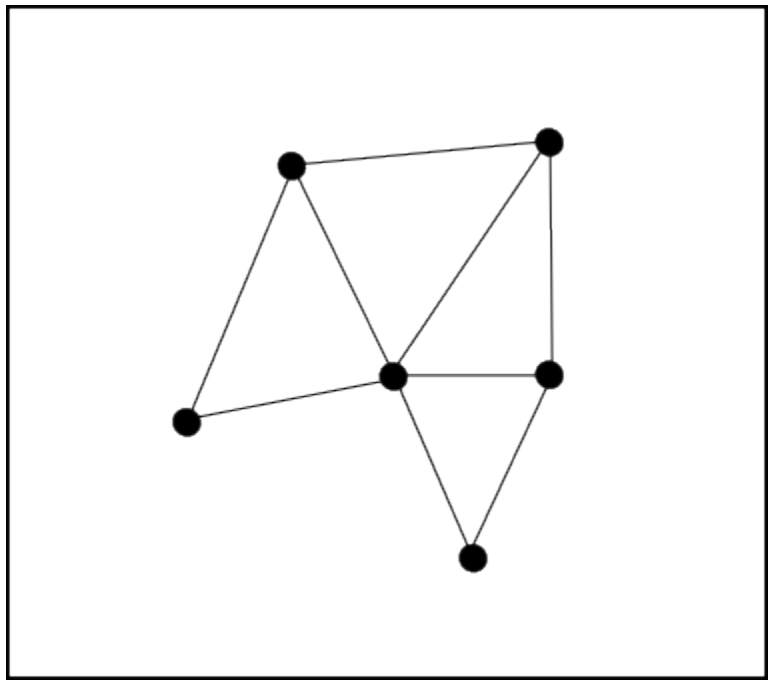


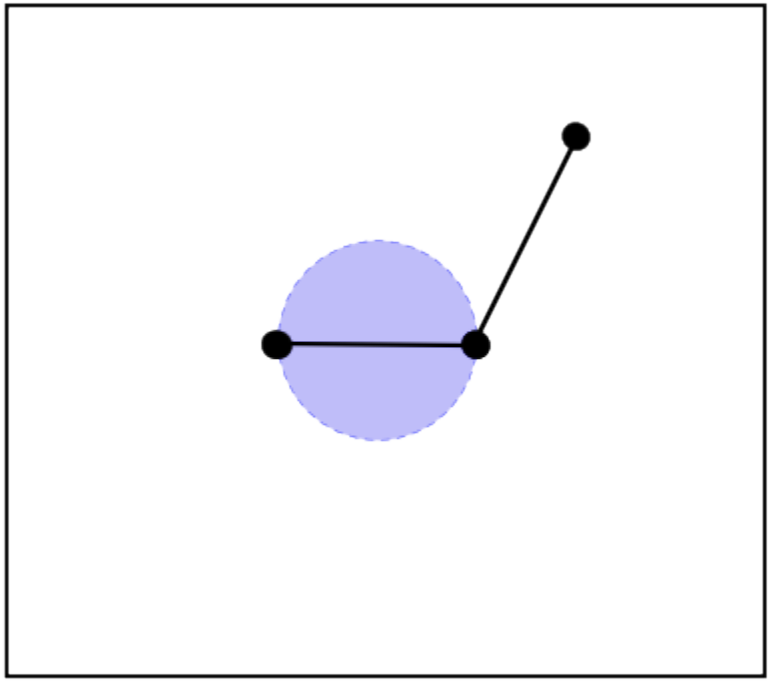


Gabriel Graphs

- Let *C(p,q)* be the circle centered on the point halfway between *p* and *q*, and with a radius of half the distance between *p* and *q.*

- The *Gabriel graph* of a set of points *V*, *RNG(V)*, is the graph that has an edge (*p,q*) if and only if the intersection of *C(p,q)* and *V* is empty.





β-Skeletons

- For β greater than or equal to 1, $U_{p,q}$ is defined as:

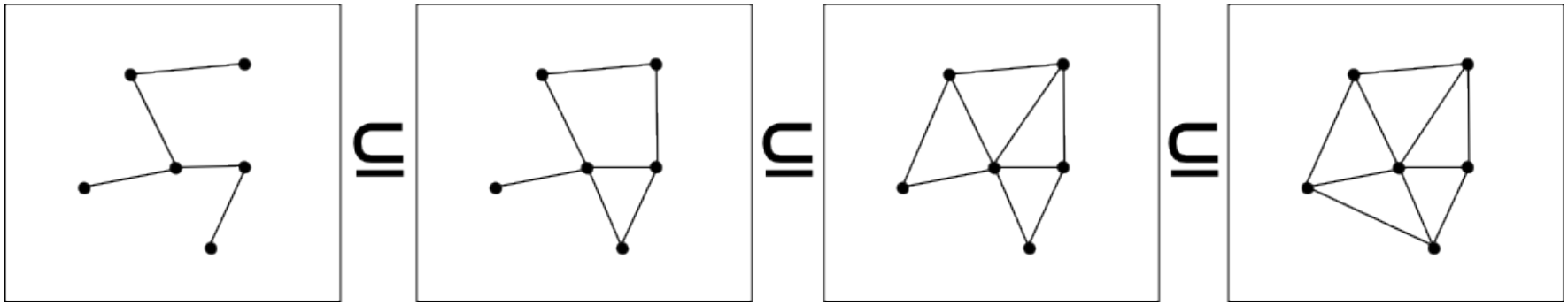
$$U_{p,q}(\beta) = B((1 - \frac{\beta}{2})p + \frac{\beta}{2}q, \frac{\beta}{2}\delta(p,q)) \cap B((1 - \frac{\beta}{2})q + \frac{\beta}{2}p, \frac{\beta}{2}\delta(p,q))$$

- The β-skeleton $G_\beta(V)$ is the graph that has an edge between p and q if and only if the intersection of $U_{p,q}$ and V is empty.
- $RNG(V) = G_2(V)$.
- $GG(V) = G_1(V)$.
- $G_m(V)$ is a subset of $G_n(V)$ for $m > n$.

A Nice Relationship

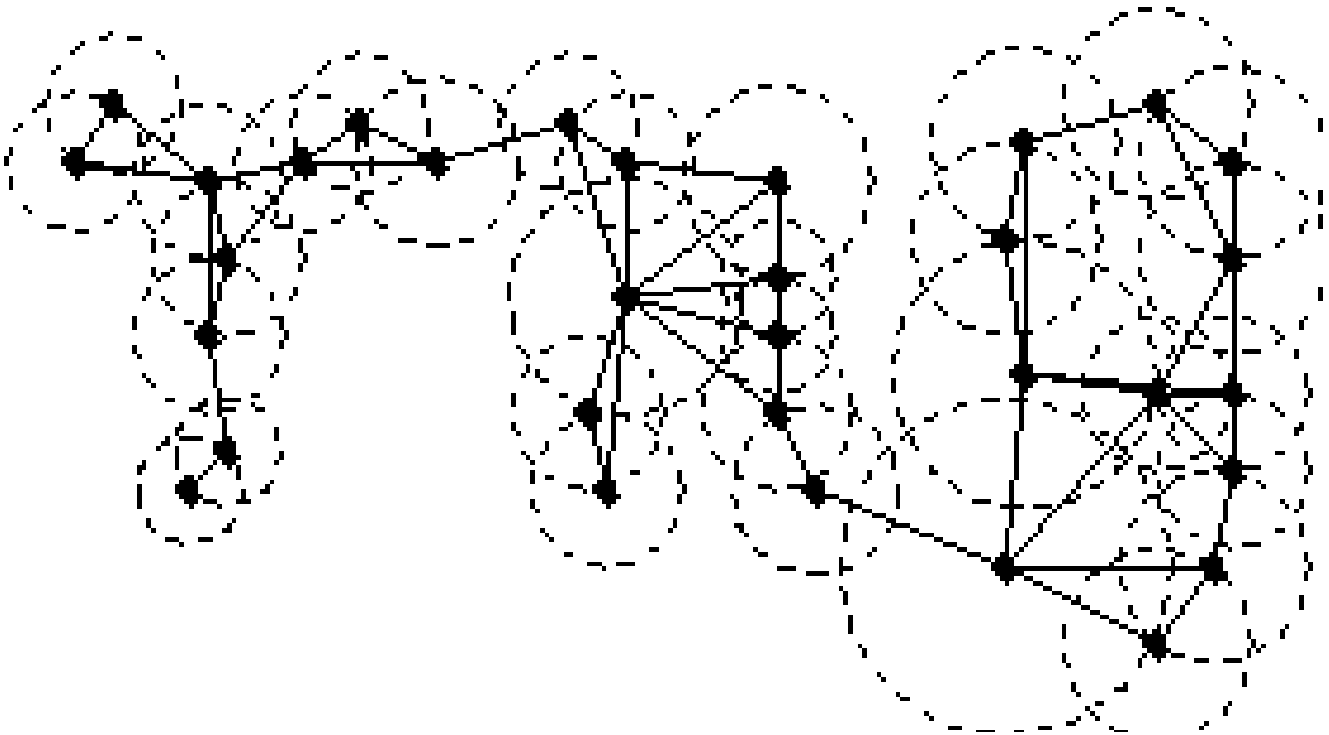
- If we also consider the *Euclidean minimum spanning tree* (which is a tree that minimizes the total edge length connecting all points) and the *Delauney triangulation* (which maximizes the minimum angle over all triangulations of a set of points), we get the following relationship:

$$EMST \subseteq RNG \subseteq GG \subseteq DT$$



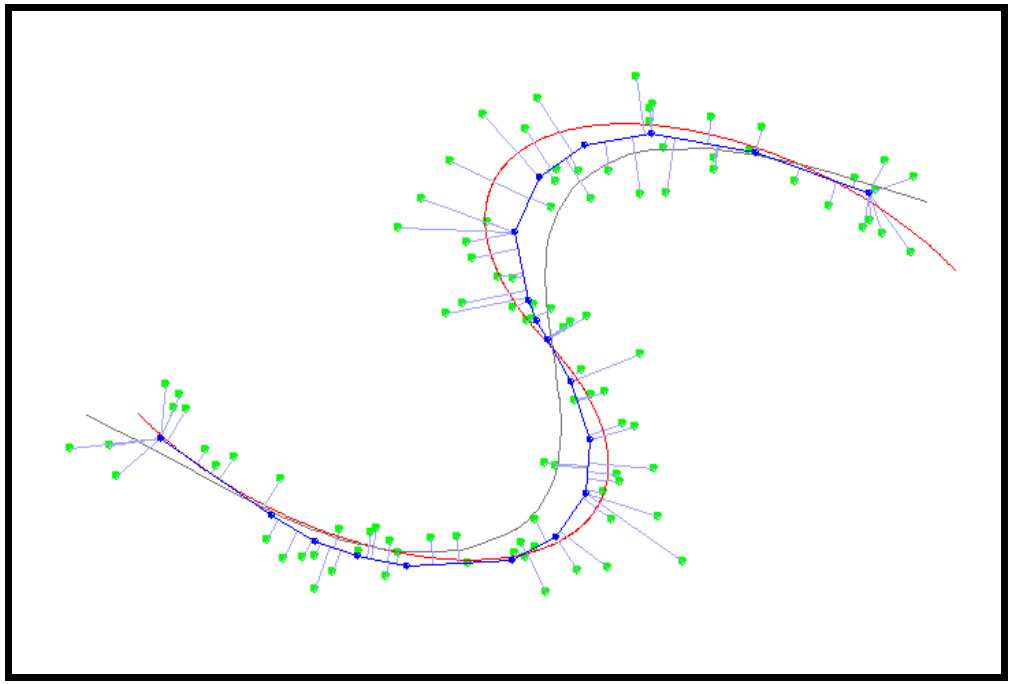
Sphere of Influence Graph

- For each point p in V, let r_p be the distance from p to the nearest other point in V, and let C_p be the the circle of radius r_p about p.
- The *sphere of influence graph* is the graph that has an edge *(p,q)* if and only if the circles C_p and C_q intersect in at least 2 places



Principal Curves

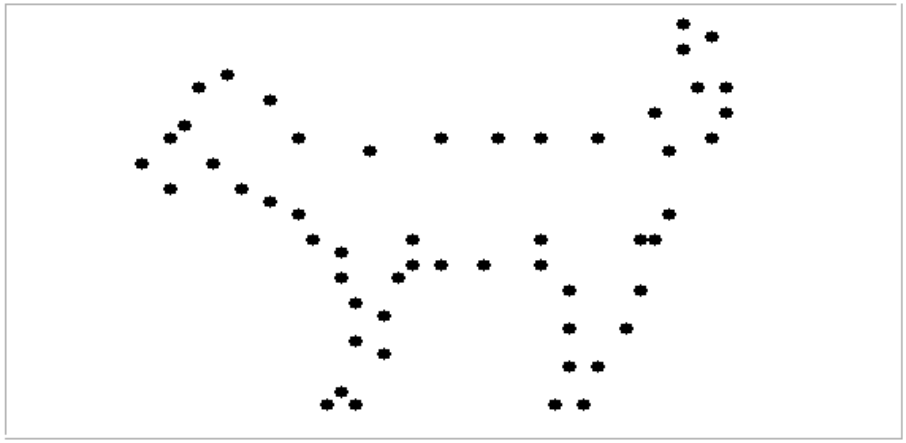
- Priciple curves are smooth curves that pass through the "middle" of a set of points (or a distribution)—"continuous curves of a given length which minimize the expected squared distance between the curve and points of the space randomly chosen according to a given distribution." [Kegl, et al, 2000]



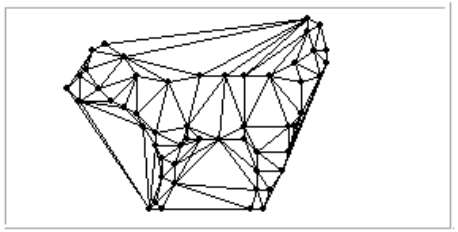
green = data; red = generator curve;
gray = Hastie/Steutzle principal curve; blue = principal curve approximation

Application: Shape Skeletons

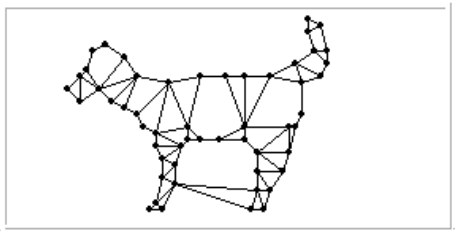
- These graphs, and the principal curve suggest several ways of finding the "shape" or the "middle" of an object.



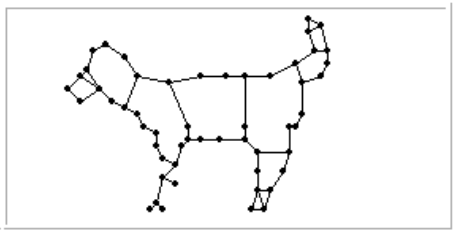
Delaunay Triangulation



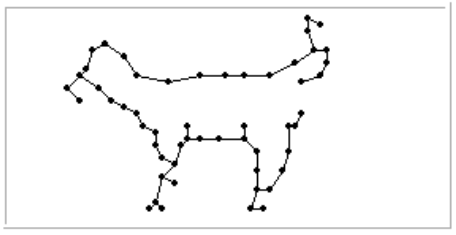
Gabriel Graph



Relative Neighbourhood Graph



Minimum Spanning Tree



What's Next?

- Implement algorithms to produce the RNG, GG, DT, etc., for a set of coplanar points.
- Implement the principal curve algorithm.
- Explore the connections between proximity graphs and the principal curve.